Comparing Range and Accuracy of Different Nerf Darts With Varying Centers of Mass

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This experiment was carried out in order to determine if there was a difference between different Nerf darts that are available for any given Nerf blaster. The purpose of this experiment was to see how shifting the center of mass of a given projectile affects its range and accuracy. The experiment consisted of three foam darts: Accustrike, Suction, and Normal darts that came with the blaster.

It was hypothesized that the dart that had its center of mass closest to its tip, the Suction dart, would have the smallest range but best accuracy. To test this hypothesis, a stand was created to hold a Nerf Eaglestrike blaster. The stand helped remove the majority of human error that would be caused if a person held the blaster. Each of the dart types had a slightly different center of mass, which was caused by having different sized tips but the same foam base. The blaster and its stand were lined up with a line found on a gym floor which was deemed to be the centerline. Each dart was assigned a number, randomized, and shot along the marked centerline. A person acting as a spotter marked the approximate landing spot with tape which was then used to measure the range and deviation of the dart from the centerline. Accustrike darts were found to have relatively the same accuracy as the Suction darts while traveling further. The normal darts traveled the furthest but were the least accurate. These findings reveal that there exists a range where the center of mass can be positioned so that the range of a dart can be maximized while maintaining relatively good accuracy. This range is predicted to exist somewhere around where the Accustrike dart's center of mass is positioned, which is closer to the tip than the Normal darts but farther from the tip than the Suction darts.

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Introduction

Would it be better to aim at long-range targets or sacrifice some of that range for accuracy? That is the tradeoff that many ballistics engineers have to consider when designing a given projectile. Ballistics is the study of projectiles, specifically the propulsion, flight, and impact of projectiles (Rafferty). Ballistics takes the basis of theoretical projectile motion and applies it to different types of munitions: bullets, rockets, etc.

The trajectories of projectiles, or objects with no means of self-propulsion, are well documented. There are equations that can accurately determine the range, final velocity, and time of the projectile while in motion. However, the basic equations mostly apply to an object where the mass is uniformly distributed throughout the object. Essentially, the basic projectile motion equations apply mostly to theoretical physics problems, not so much to real-life physics problems where there are other factors such as drag, altitude, and the shape of the projectile. In this experiment, the shape of the projectile is considered and tested.

Bullets and other ballistics are specifically shaped to cut through the air with speed and accuracy (Alioto). Different shapes and materials give bullets special aerodynamic capabilities that enhance their performance. These different shapes shift where the center of mass is in the object. This is something that the basic projectile motion equations do not take into account; as mentioned before, they assume that the mass is uniformly distributed and that the center of mass is in the exact middle of the object. In ballistics, however, the center of mass is one of the major factors taken into account as it can help determine the range and accuracy of the projectile being launched.

This experiment looks explicitly at how the center of mass affects the range and accuracy of projectiles. A dart's accuracy was determined by how far it deviated from a set centerline.

The center of mass is defined as the mean position of all the parts of a system (What is Center of Mass). As mentioned before, the basic projectile motion equations look at objects where the center of mass is at a position called the centroid. This experiment uses nerf darts to look at how changing the center of mass will affect its range and center of gravity. The three types of darts used in this experiment all have the same foam base. The difference between them is their tips which come in different shapes and sizes. The slightly heavier tips shift the center of the mass toward the front of the dart. For example, the Suction darts are slightly heavier than the Normal darts and the Accustrike darts; this is due to its tip being heavier, leading to their center of mass being closer to the front.

In order to minimize confounding effects, like air resistance and human error in aim, the blaster shot darts over a straight line from a fixed mount, and all trials were completed in a closed gym. Although blasters will have built in error in real-world applications, data would not be reliable if darts were shot from different angles, heights, and settings. The darts were shot at a height that an average person would hold the blaster roughly five feet above the ground. To determine a dart's accuracy, its deviation from the centerline was measured. A dart's range was found by measuring the straight distance from the mount. And the dart's accuracy was based on how far the dart's landing spot deviated away from the centerline.

The most important part of this experiment is to see if there is an optimal position where range can be maximized while maintaining accuracy. The findings from this experiment could help Nerf create better and more effective darts. On a larger scale, these findings could be applied to ballistics research in general from bullets to large missiles. A projectile whose center of mass maximizes its range will most likely not be that accurate. It all comes back to that tradeoff between accuracy and range.

Review of Literature

In this experiment, projectile motion was the major topic as the Nerf darts were projectiles from the blaster. Projectile motion deals with the flight of projectiles, or objects with no means of self-propulsion, and their trajectory, or flight path. The trajectory of a projectile usually follows a parabolic path. In instances where characteristics of the projectile are not changed, such as a ball or any spherical object, initial speed and the angle a projectile was launched are the main factors affecting the motion, as seen in figure 1. The equation $R = \frac{v_0^2 \sin 2\theta}{g}$ determines the range of a projectile. As the initial speed of the projectile increases, so does its range. Varying angles of the launch point also affect the range with 45° being the optimal angle (University Physics Volume 1).

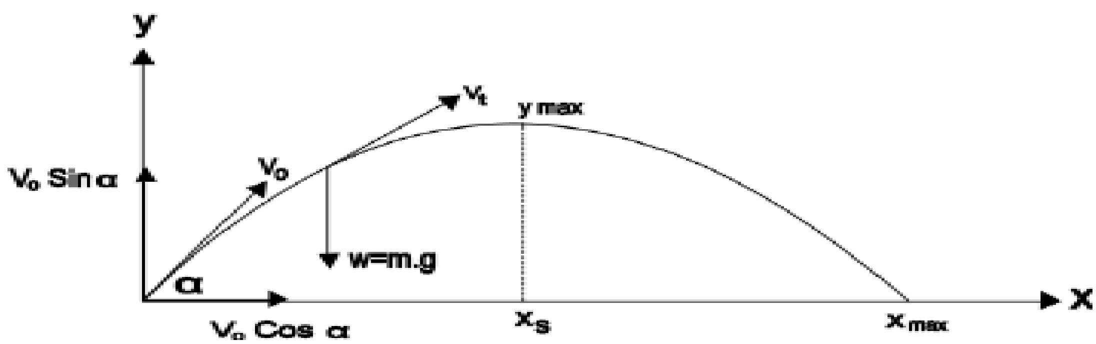


Figure 1. Projectile Motion Diagram

Projectile Motion. Sourced from: “Trajectory.” The Physics of Air Rifles, http://ffden-2.phys.uaf.edu/webproj/212_spring_2019/Zion_Alioto/zion_alioto/Page4.html.

Because most ballistics and nerf darts have an asymmetrical shape, their center of mass is shifted away from the middle. The center of mass of an object is the average of all the parts in the system (What is Center of Mass). This unique position follows where most of the mass is distributed in an object. For smaller projectiles, such as the nerf darts used in this experiment, one can attempt to balance the dart on one of their fingers. They would find that there is a certain spot on the dart where they can rest their finger under and the dart will be balanced. This spot is the center of mass on an object. The basic projectile motion equations deal with objects with a uniformly distributed mass. For these objects, the center of mass is located at their centroid or their geometric center.

But most ballistics deal with projectiles of varying shapes with different centers of masses. For example, the bullet in figure 2 has most of its mass concentrated towards its rear. Due to this, the center of mass is pulled towards the rear. This is because bullets are fired from guns that can push on the flat surface of the bullet's rear.

However, this is the opposite for nerf darts where the center of mass is actually towards the front. The mass is evenly distributed except for the plastic tip which weighs slightly heavier than the foam body. Unlike a bullet, a nerf dart is fired from a Nerf blaster. The foam base of a dart has a hole which is needed in order for the blaster to launch it out.

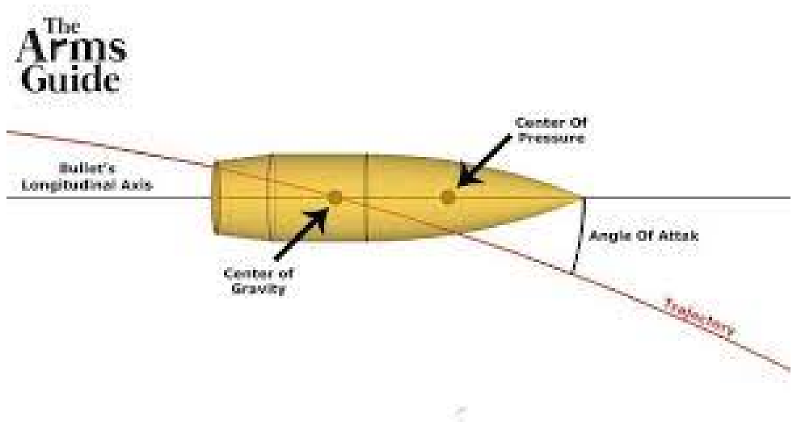


Figure 2. Center of Mass and Trajectory of Common Ballistics.

Matt. “Center of Gravity and Pressure of Bullet.” The Arms Guide, 22 June 2017, https://noebulletmolds.com/smf/index.php?topic=2729.0. Accessed 17 Mar. 2022.

The trajectory of the projectile will follow a parabolic path as demonstrated by the red trajectory line in the above image. The direction of the tip of the bullet is determined by the trajectory of the center of gravity (Ling). This is due to how gravity, being the main external force, acts on a projectile and Newton's second law of motion ($f = ma$).. Force is the product of mass and acceleration; and since the acceleration of gravity is uniform throughout, the region where the mass is most concentrated, this being the center of mass, will have the largest force acting on it. Therefore, a projectile’s center of mass must have some effect on its range and accuracy.

As shown in figure 3, the center of mass for a nerf dart is mostly towards the front due to the heavier plastic tip. Like with a normal bullet, the center of gravity has a parabolic trajectory that determines the angle at which the dart is pointing. Therefore, by manipulating where this center of mass is, the trajectory of the entire dart can be altered which could lead to either a higher or lower range.

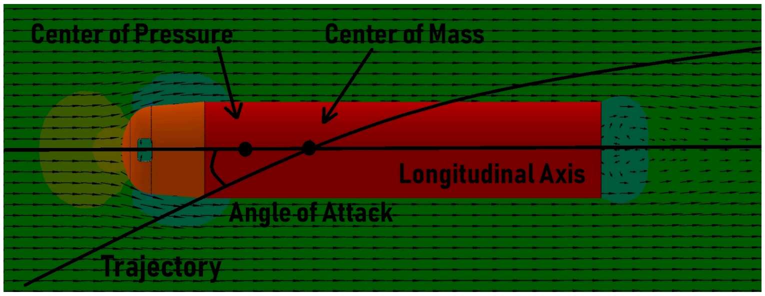


Figure 3. Center of mass and trajectory of a nerf dart. Alioto, Zion."AERODYNAMICS." *The Physics of Air Rifles*, 2019, http://ffden-2.phys.uaf.edu/webproj/212_spring_2019/Zion_Alioto/zion_alioto/Page3.html.

When it comes to ballistics, the shape, and aerodynamic properties are also taken into account. Aerodynamics is the study of the motion of objects through the air which includes projectiles. Certain shapes allow a projectile to cut through the air with more efficiency. Although this experiment does not deal with the aerodynamic properties of the Nerf dart, it still does play a role. Drag, or the force that opposes an object in motion due to air resistance, is given by the equation $F_D = \frac{1}{2}C_d Apv^2$ where F_D stands for drag and A stands for cross sectional area (The Drag Equation). As seen from the equation, the force of drag and a flying object's cross sectional area is proportional. A greater drag force means more opposing force to the projectile, which would therefore decrease its range. The Suction dart had the greatest cross-sectional area which meant it would be subject to the greatest drag force. Due to this, it was hypothesized that the suction dart would have the shortest range out of the three darts. That said, however, heavier objects tend to be more stable in the air. Because of this, it was hypothesized that the suction dart would be the most accurate dart out of the three.

Problem Statement

Problem:

To discover how the position of the center of mass affects the range and accuracy of a nerf dart.

Hypothesis:

Moving the center of mass closer to the tip will result in less range but better accuracy for the dart.

Data Measured:

The explanatory variable was the type of dart used which is distinguished by where the center of mass is located relative to the dart. The center of mass was located somewhere between the middle and tip of the dart. This depends on the dart's weight; for example, a heavier dart had a center of mass closer to the tip. The response variable will be the range of the dart from the tip of the blaster to where it first hits the ground and the deviation which is measured by how far the dart landed away from a set centerline. Both response variables were measured in inches rounded to the nearest quarter inch. Three different types of darts will be used and each will be randomly assigned the trial it gets launched on. Forty-five trials were run for each dart; the mean range and deviation for each dart was compared to see whether there is a difference.

In this experiment, box plots were first used to determine whether there is a significant difference in the range and deviation between the darts. If there were no clear difference among all three darts, then an ANOVA test would have been used as multiple means will be compared. But since one dart had significantly different data from the other two, then a two sample *t*-test was necessary as two means will be compared.

Experimental Design

Materials:

Nerf Elite 2.0 Eagle Point RD 8
(40) Nerf Elite Darts
(40) Nerf N-Strike Suction Nerf Darts
(40) Nerf Dart Zone Darts

Tape Measure (20')
Stand (Refer to Appendix B)
Tape
Sharpie/Pen

Procedure:

1. Using a TI Nspire, create a column of 1-135.
2. Randomize 135 trials in the next column.
3. Assign 1-45 to Normal, 46-90 to Accustrike, and 91-135 to Suction.
4. Use tape to create a centerline in order to measure the dart's accuracy. Have the shorter edge of the tape be the zero line that is perpendicular to where the blaster is facing. Have the right or left edge of the tape be the zero line that is parallel to the blaster. Extend the centerline to roughly 90 feet.
5. Line up the mount with the tape, making sure the barrel of the blaster is lined up with the centerline. Check that the tip of the dart in the blaster lines up with the zero line. (See figure 7).
6. Mount the Nerf blaster so that the vertical center of the dart is 5' off the ground. (See figure 4 and 5)
7. Unlock the barrel and load it with eight darts in their trial order.
8. Move the blaster's orange slide back.
9. Press the trigger to launch the dart.
10. Have a person act as a spotter and place a piece of tape on the approximate location of the dart's landing.
11. Repeat steps 8-10 until the eight trials in the barrel are completed.
12. Repeat steps 7-10 for forty-five darts.
13. Use the tape measure to measure the horizontal distance from the edge of the blaster to where the tape is marked.
14. Measure from the centerline to the middle of the tape to get the deviation.
15. Repeat steps 7-14 for the rest of the trials.

<u>Diagrams</u>:

Figure 4. Experiment Set-Up

Figure 4 above shows the design of the experiment. A dart will be shot 5' above the ground. It will land at a certain spot and the spotter will mark the approximate location.

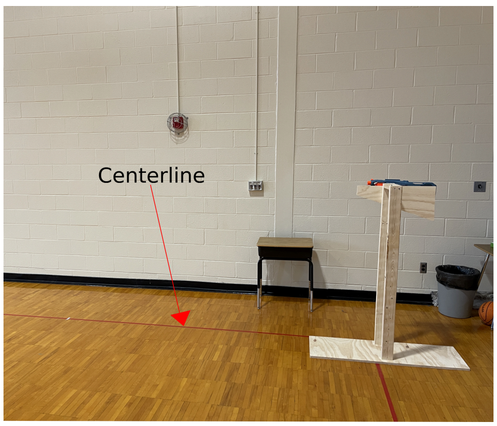


Figure 5. Experiment Overview

The figure above clarifies the depiction as shown in figure 4. The stand holds the blaster in place, letting the dart travel from the same point every time.

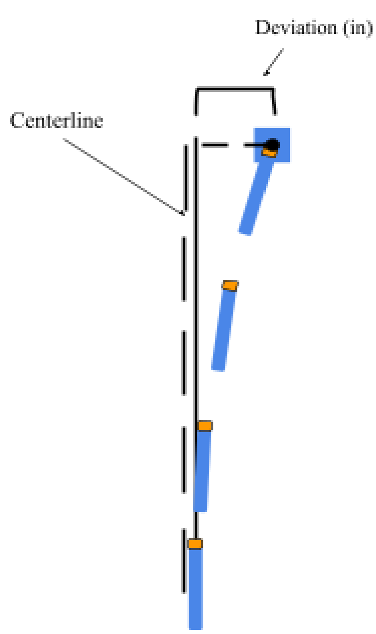


Figure 6. Experiment Set-Up

Figure 6 above demonstrates what is being measured in the experiment. The vertical line on the figure represents the range and the horizontal line represents the deviation of the dart. The approximate place where the dart will land is marked by the center of the tape.

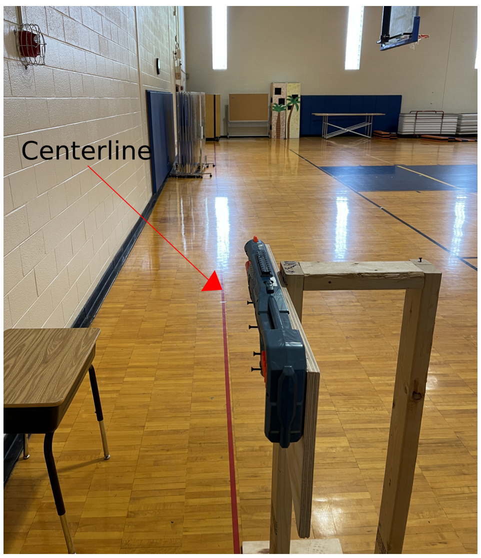


Figure 7. Experiment Setup

Figure 7 above shows the nerf blaster mounted onto the stand. The person shooting should line up a piece of wood on the right and push the blaster forward and against the nails when shooting. The bottom of the image demonstrates how the blaster is lined up with the red line.

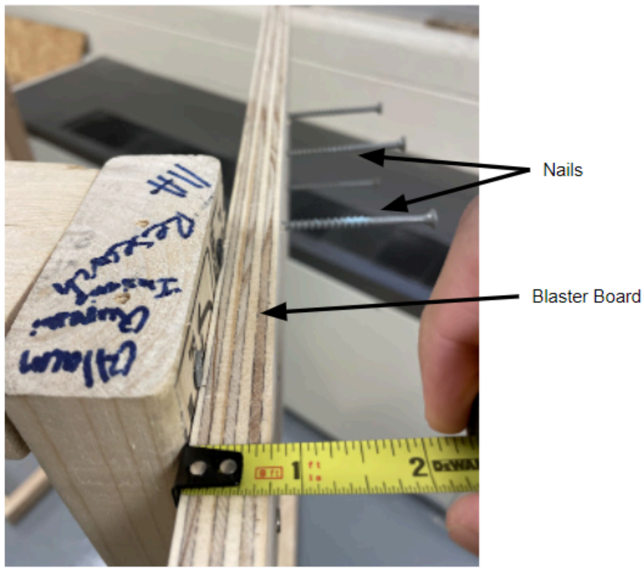


Figure 8. Measurement of the Center of the Blaster

It was determined that the center of the blaster, or the center of the dart when it is loading, is 1.5” away from the edge of the 2x4.

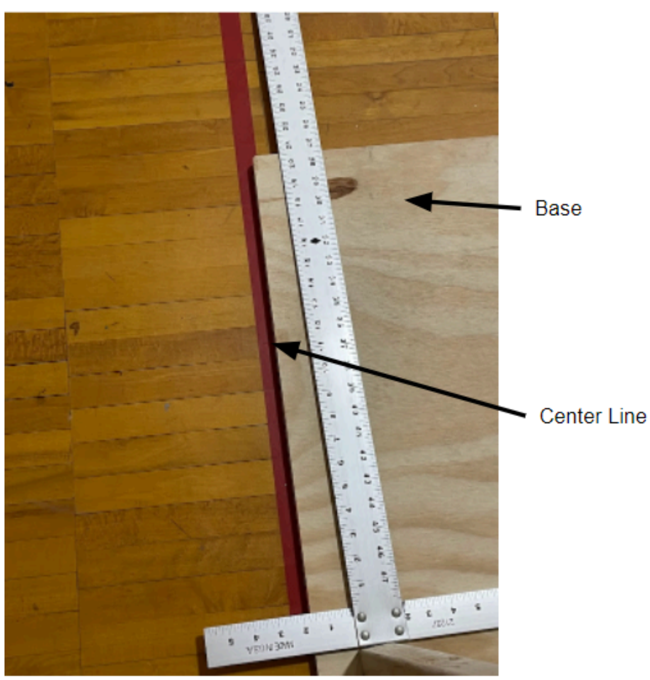


Figure 9. Stand and Centerline

The edge of the red line was used as the centerline. The edge of the base is 1.75” from the edge of the 2x4. Because the center of the blaster was 1.25” away from the 2x4, the edge of the base should be 0.25” to the left of the centerline.

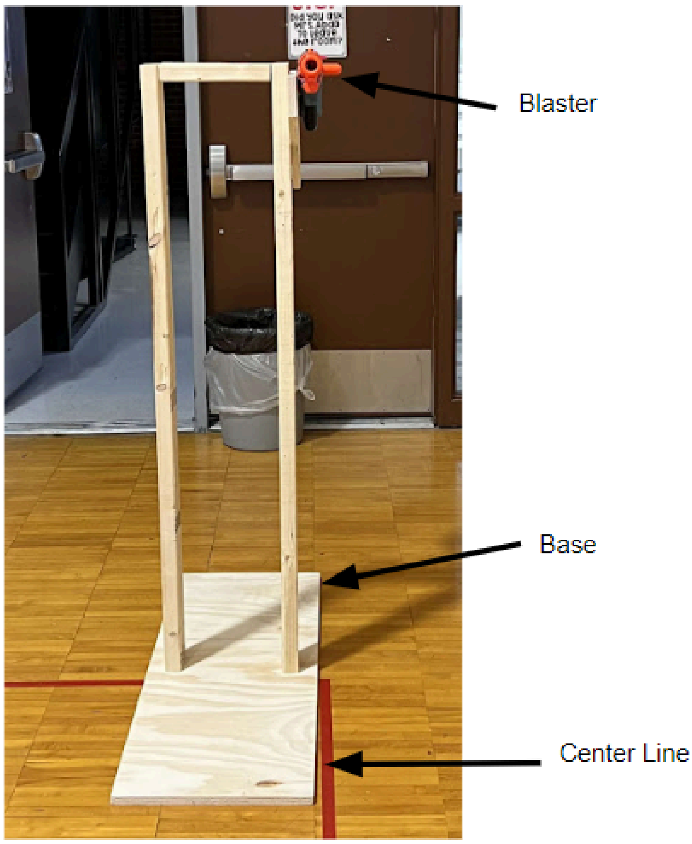


Figure 10. Finished Stand Configuration

Figure 10 above shows a front view of the blaster and stand. The Nerf blaster is mounted using screws. The blaster is mounted by being placed onto semi-fastened screws. The long 2x4 was fastened to the smaller one using screws on both sides. The bottom of the 2x4 is fastened to the base such that the legs are in the center. A side piece of plywood was fastened to one of the 2x4 in order to mount the blaster.

Figure 11. Nail Position on Stand

Four nails were drilled to hold the blaster up in spots where the blaster would not have much room to move around. For the experiment the blaster was pushed forward and toward the wood to ensure the trajectory of each dart was similar to each other.

Data and Observations

Table 1
Data of Trials Collected

All Trials			
Dart Type	**Trial #**	**Range(in)**	**Deviation (in)**
A	1	397.50	0.50
N	2	443.00	52.50
S	3	415.00	0.50
A	4	342.00	5.50
A	5	327.00	3.00
S	6	400.00	5.00
S	7	329.00	0.00
A	8	417.00	5.00
N	9	457.00	2.50
N	10	625.00	7.50
A	11	393.50	1.50
S	12	331.00	3.25
N	13	358.00	17.00
A	14	407.00	8.25
N	15	313.00	5.50
N	16	402.00	3.00
A	17	402.00	0.00
N	18	629.00	10.50
N	19	366.00	6.00
N	20	398.00	9.00
A	21	424.00	17.50
S	22	336.50	0.00
S	23	347.00	8.00
A	24	451.50	4.50
N	25	385.00	32.50
S	26	370.50	9.50
N	27	531.00	20.00

Dart Type	Trial #	Range (in)	Deviation (in)
A	28	379.50	1.00
A	29	391.50	0.50
S	30	351.50	3.50
A	31	362.00	1.50
N	32	615.50	15.00
N	33	382.50	12.50
S	34	340.00	9.00
S	35	259.00	1.50
A	36	386.50	0.50
A	37	376.50	3.50
N	38	406.75	18.00
N	39	375.25	20.00
A	40	355.50	2.00
N	41	416.00	19.50
A	42	395.00	3.50
N	43	371.00	44.50
A	44	334.25	0.00
S	45	312.50	0.50
N	46	439.50	8.25
A	47	352.00	0.50
N	48	455.25	5.00
A	49	410.00	4.00
A	50	361.00	3.50
N	51	588.00	12.50
S	52	332.00	3.25
S	53	295.50	5.00
A	54	374.75	1.50
S	55	342.50	6.25
N	56	406.50	2.50
A	57	378.00	2.50
S	58	419.50	4.00

Dart Type	Trial #	Range (in)	Deviation (in)
N	59	381.00	7.25
A	60	403.00	12.50
N	61	575.00	17.75
A	62	370.00	0.50
A	63	344.50	3.50
S	64	342.00	10.50
A	65	386.50	4.00
A	66	388.00	0.50
N	67	396.00	12.50
A	68	411.00	2.00
A	69	337.50	3.00
S	70	284.25	6.50
S	71	331.50	4.00
S	72	401.50	14.75
N	73	453.75	4.50
S	74	364.50	0.00
S	75	321.00	2.50
N	76	465.50	14.00
S	77	314.00	0.50
A	78	397.00	7.00
N	79	432.00	6.25
S	80	386.00	7.50
S	81	321.00	4.00
A	82	367.00	0.50
A	83	377.50	4.00
N	84	613.00	43.50
N	85	402.50	3.50
A	86	348.00	1.00
S	87	330.50	8.00
A	88	432.50	6.50
S	89	358.00	3.50

Dart Type	Trial #	Range (in)	Deviation (in)
N	90	551.00	6.00
N	91	494.50	1.00
A	92	400.50	5.00
S	93	268.50	0.50
A	94	362.50	0.50
S	95	412.00	2.50
S	96	352.50	4.00
A	97	335.00	1.50
A	98	354.00	1.00
A	99	382.00	3.00
N	100	578.00	36.25
A	101	365.00	2.50
S	102	340.50	5.50
A	103	394.50	6.50
S	104	306.00	0.50
N	105	488.00	31.50
A	106	393.00	5.50
A	107	399.00	19.75
N	108	417.00	38.00
N	109	493.00	34.50
A	110	398.00	1.00
N	111	371.00	50.00
S	112	280.00	18.00
A	113	420.50	3.25
A	114	413.00	4.50
N	115	524.00	41.50
S	116	317.00	11.00
S	117	314.00	11.00
S	118	334.50	6.50
S	119	309.00	14.25
N	120	514.25	53.00

Dart Type	Trial #	Range (in)	Deviation (in)
N	121	394.00	21.00
S	122	328.50	8.50
S	123	328.50	4.00
N	124	441.00	12.00
N	125	409.50	9.00
S	126	365.50	0.00
N	127	469.00	38.50
S	128	328.50	1.50
N	129	539.50	9.75
S	130	296.50	9.75
S	131	451.00	5.00
S	132	372.50	2.50
S	133	362.00	0.50
S	134	332.00	4.25
S	135	320.50	2.50

Table 1 shows the distance and deviation from the centerline of each of the 135 trial darts in inches. Dart Type "N" are Normal darts, "A" are Accustrike darts, and "S" are Suction darts.

<u>Experimental Process:</u>

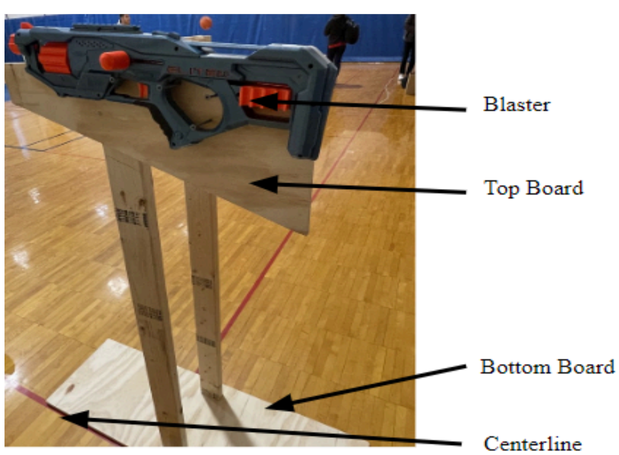


Figure 12. Blaster Mount

Figure 12 shows how the blaster was mounted. The person using the blaster lined up the bottom board of the mount so that the front of it sat on the right edge of the red line and the middle of the board lined up with the middle of the red line. The blaster was pushed forward and to the right into the top board to keep the barrel straight.

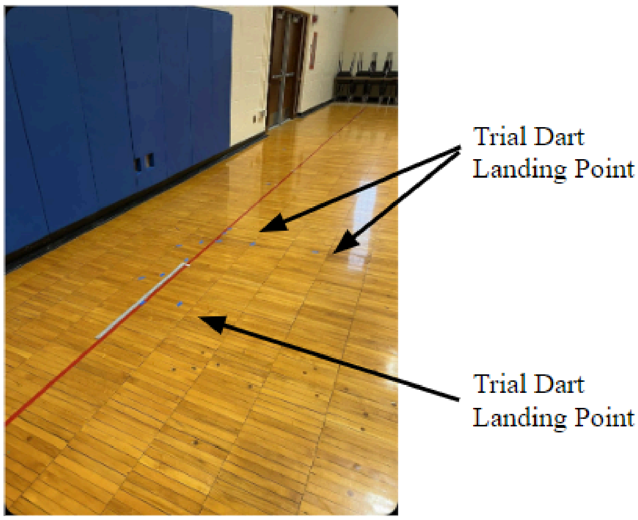


Figure 13. Data Points

Figure 13 above shows an example of roughly 20 trials on the gym floor, all of which were measured for their range and deviation.

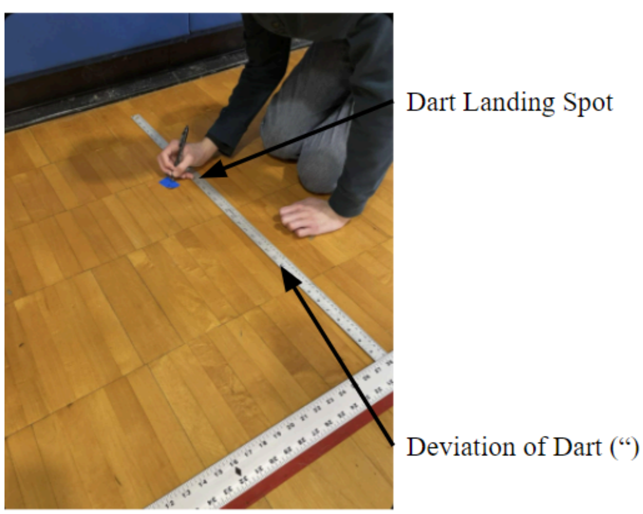


Figure 14. Data Collection.

Figure 14 above shows a group member that measured and recorded, on the masking tape where the dart landed, the range and deviation of a certain dart.

Table 2
Unusual Trial Observations

Trial	Observations
18, 120	Trials 18 and 120, both of which were Normal darts, curved through the air more extremely than any other dart. They produced the largest range and deviation, respectively.
116-119, 130-135	Suction darts generally landed shorter, but veered less than the other two dart types. Suction darts were the most consistent, producing only one outlier between its range and deviation. Unlike Normal and Accustrike darts, Suction darts did not hit a spotter, as their landing points were fairly predictable.
9-10, 13, 15-16, 18-20	Normal darts were the most unpredictable. They produced a range for range and deviation so wide that there were no outliers, the dart moves extremely by nature.
21, 60, 107	These trials were the outliers for the range of Accustrike darts. Aside from the outliers, Accustrike darts generally went farther than Suction darts and deviated less than Normal darts.

Data Analysis and Interpretation

In this experiment, data for range and accuracy of various nerf darts were quantitative and continuous, as they were measured in inches to the nearest quarter. To randomize the 45 trial shots per dart, a column of 1 through 135 was made on a spreadsheet page on a TI-Nspire calculator. The 135 trials were split up into three groups: 1-45 were designated as Normal darts, 46-90 were Accustrike darts, and 91-135 were Suction darts. The calculator then randomized 1-135 in the next column. Trials were randomized so that errors in the experiment, such as blaster malfunction, or confounding variables, such as drag, would affect all darts equally. Trials were replicated because the purpose of the experiment is to compare the effects on the flight of darts with different centers of masses. If the experiment was set up a slightly different way each day, the causes of the differences in the distributions of range and deviation of the darts would not be clear. They could either be attributed to different centers of masses, or because the blaster shot at a slightly different angle each day.

To ensure the data was collected consistently and is reliable, the blaster stand was set up a specific way the first day, and was replicated as closely as possible the next few days. As seen in the diagrams in Experimental Design, because the center of the blaster was 1.5” away from the edge of the 2x4, and the edge of the base was 1.75” away from the 2x4, the edge of the base was 0.25” to the left of a centerline. If the stand was not set up the same way, it could be shooting darts at an angle, creating inconsistent data for accuracy. Once the stand was in place, the blaster was mounted onto the screws. To ensure that the blaster was shooting completely straight each time, the person shooting the darts pushed the blaster all the way forward and to the right into the wood. Once the blaster was ready, 135 blue pieces of masking tape were labeled with its trial number and dart type according to the randomized trials in the calculator. Since the barrel to the blaster had 8 spots, 8 trials were done at a time. The correct order of darts to the next 8 trials were loaded into the blaster. One group member shot the darts according to the correct order of trials, while the other two carefully watched where the darts landed and marked their spots with the respective labels. If a dart hit a team member while they were watching where the dart landed, or hit the wall of the gym, that trial, with the same dart, was immediately redone. Darts hitting the wall would be obvious outliers to accuracy, and darts' landing points that hit a spotter would not be assumed or guessed. After forty-five darts were shot and marked, each darts' range and deviation from the centerline were measured in inches. The data was then entered into a spreadsheet. Because the Normal darts are the most common, the other two darts' range and deviation were compared to those of the Normal darts.

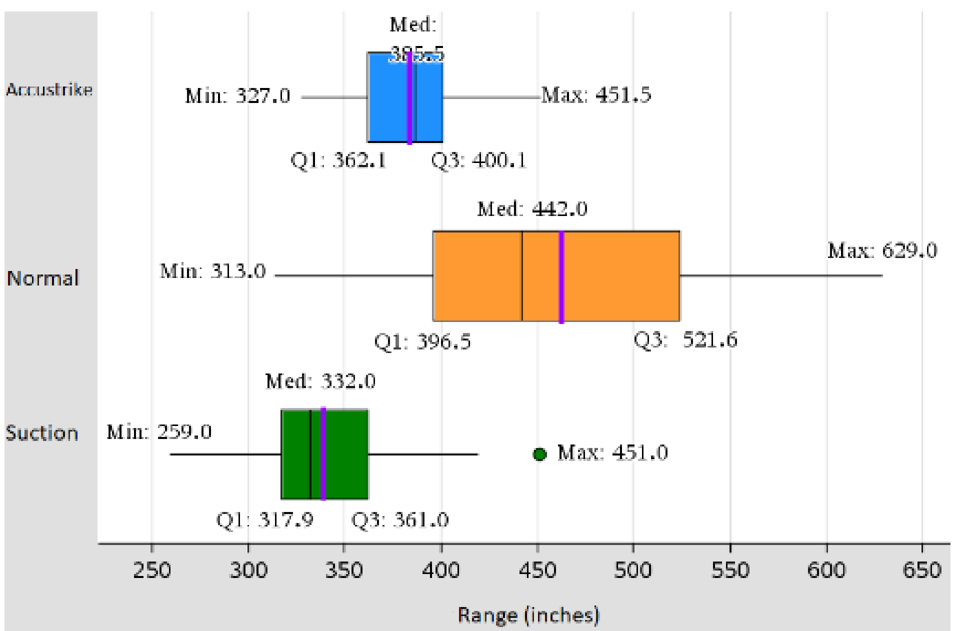


Figure 15. Box Plots of Range of Darts

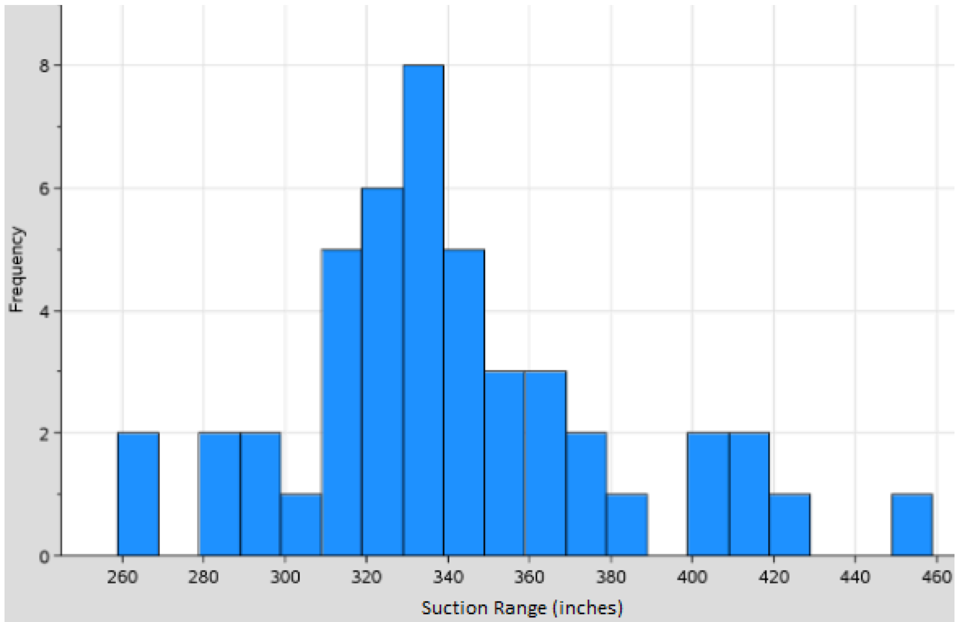


Figure 16. Histogram for Suction Dart Range

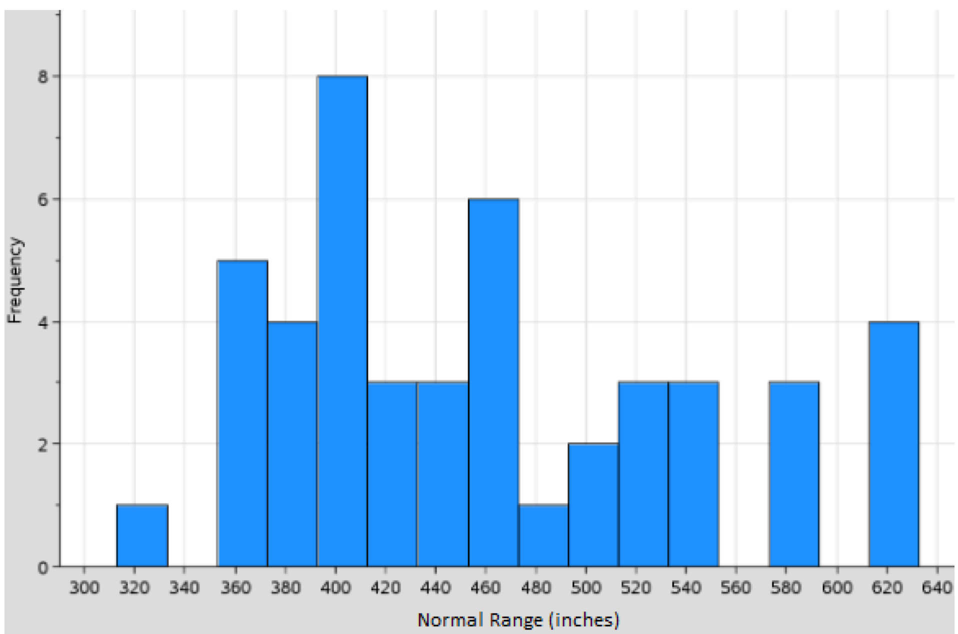


Figure 17. Histogram for Normal Dart Range

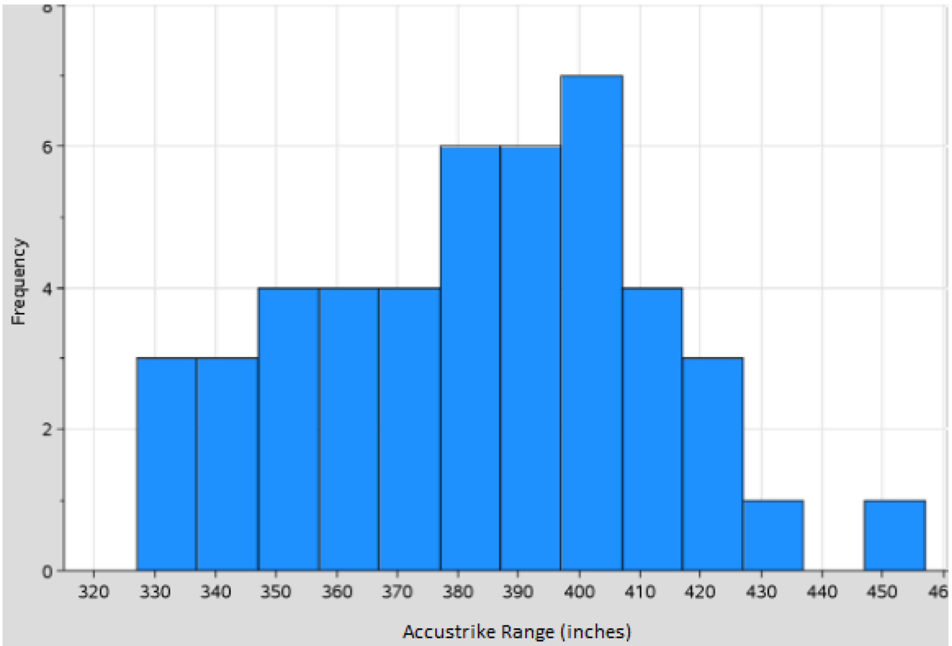


Figure 18. Histogram for Accustrike Dart Range

The center of the Accustrike range distribution is about 390” . The distribution, as shown in figure 18, is roughly normal with no outliers and has the largest range of values of 124.5”. The center for the Suction range distribution is at around 340” and has a bigger range of values than the Accustrike set but a smaller range of values than the Normal darts. Figure 16 shows that the distribution of the Suction dart range is also approximately normal but with one high outlier. The Normal distribution of ranges has the highest median at around 440” and the greatest range of values when compared to the others. This distribution seems right skewed and there are no outliers based on Figure 17.

As shown in the box plots in figure 15, there is a clear difference in the sample distribution of range for the Normal darts as compared to the Accustrike and Suction darts. The third quartile of the Normal darts is higher than the maximum for both the other types of darts by around 70”. Of all the Normal darts, the median (422”) is greater than the third quartile of both the other darts. Due to this, it was concluded that there was a significant difference in the range of Normal darts when compared to the Accustrike and Suction darts. When comparing Accustrike and Suction, however, there is no clear difference. Although Accustrike had a higher median and higher quartiles, the Suction distribution had a greater range of values. In order to come to a sound conclusion, a two-sample *t* test was done to determine whether there truly is a difference in the means.

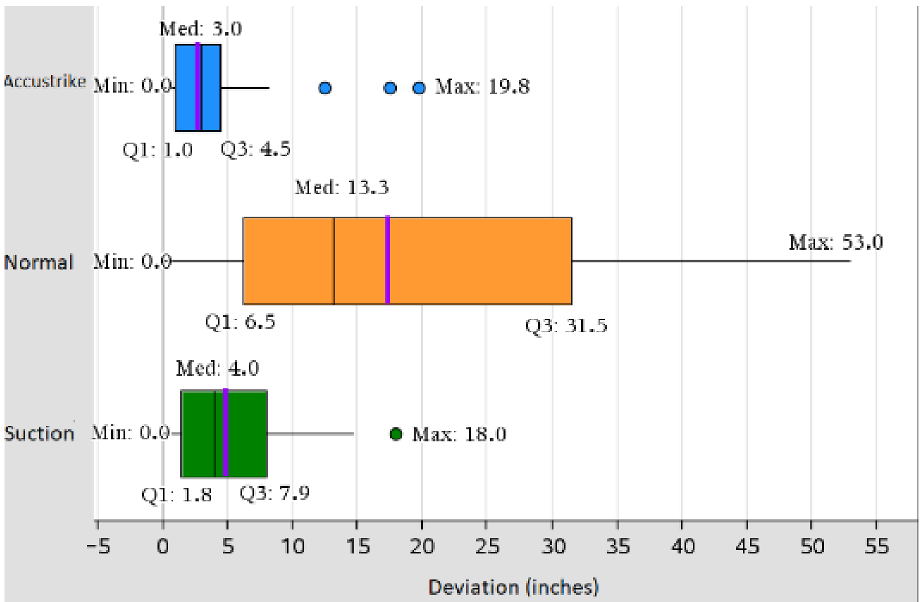


Figure 19. Box Plots of Deviation from the Centerline

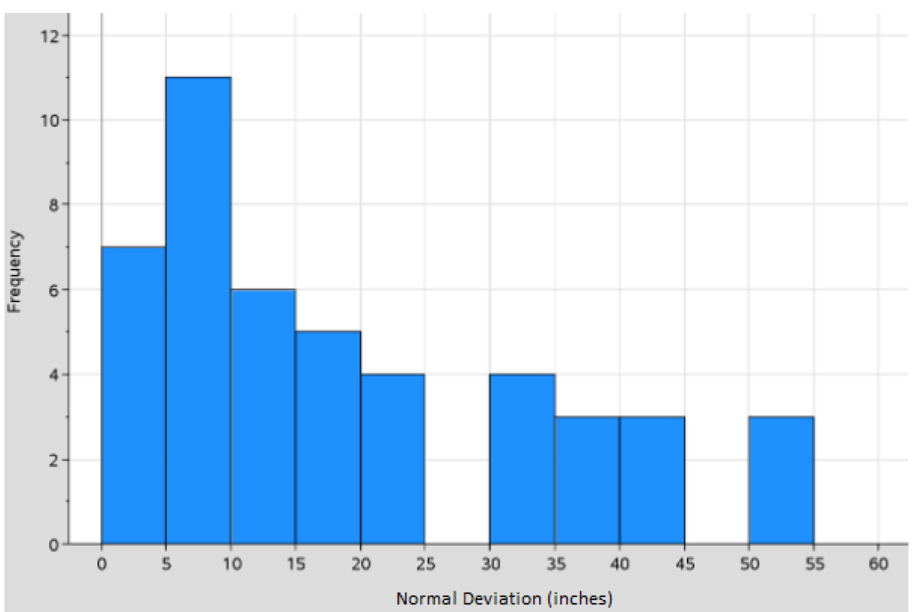


Figure 20. Histogram for Normal Dart Deviation

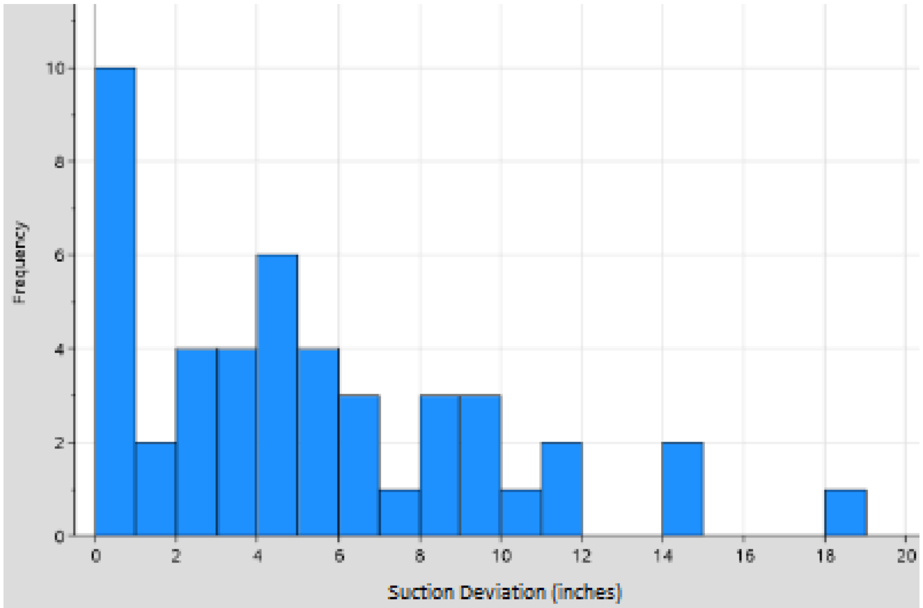


Figure 21. Histogram for Suction Dart Deviation

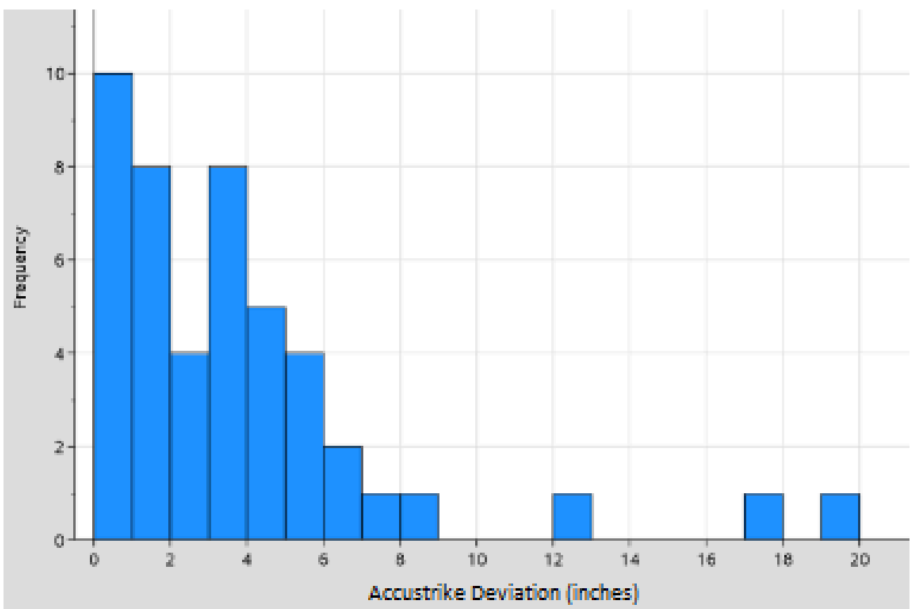


Figure 22. Histogram for Accustrike Dart Deviation

Figures 20, 21, and 22 show that all three sample distributions for deviation are right-skewed. Accustrike had 3 high outliers and Suction had one high outlier. Every histogram has some gaps in the data with the deviation histograms for Accustrike and Suction having the largest gaps. Accustrike had the smallest mean and median followed by Suction and then normal. Much like before, it was concluded that there is a significant difference in deviation from the centerline when comparing the Normal darts to the other darts just from looking at the box plots. Both the median and mean are at or around the maximum of the Suction and Accustrike darts. Again, however, there is a much less clear difference between the Suction darts and the Accustrike darts. The means and medians between the deviations of these darts are roughly similar. That said, however, 75% of the Suction darts had more deviation than almost all of the Accustrike darts, not including the high outliers. Another two-sample *t* test was done to determine whether there truly is a difference.

As shown in the box plots in Figure 19, there is again a clear difference in the sample distribution of deviations for the Normal darts as compared to the Accustrike and Suction darts. The third quartile is again much higher than the maximum values for both the Accustrike and Suction darts. In fact, the median of the Normal darts is greater than the maximum of the Accustrike darts not including the outliers. Due to this, it was concluded that there is a significant difference in the deviation of Normal darts when compared to the Accustrike and Suction darts. When comparing Accustrike and Suction, however, there was no clear difference. The Accustrike darts had a lower median and lower quartiles when compared to the Suction darts. Suction, with outliers considered, had a slightly larger range of deviation than Suction but far less than the range for the Normal darts. However, its interquartile range of 3.5” is slightly smaller than Suction (6.1”) and far less than Normal (25”). Another two sample *t* test was done to determine whether or not there exists significant evidence for a difference in the true mean deviation between the Accustrike and Suction darts.

A two-sample *t* test for means was deemed appropriate because the means from two independent populations were tested in order to determine whether there was a significant difference. The null hypothesis states that there is no difference in the mean range of Accustrike and Suction darts. The alternative hypothesis states that there is a significant difference in mean ranges between the Accustrike and Suction darts.

All of the conditions and assumptions were appropriate to run a two-sample *t* test. The random condition was met as the trial order was randomly generated using a calculator. Each dart had an equal chance of being selected to be used in each trial. This ensures that if there was a problem with the experiment, such as the blaster not functioning as intended for a certain number of trials, it would affect the data of all the darts evenly.

The normal condition requires the shape of the sample distribution to be approximately normal and show no major skewness and outliers. Although there were outliers and skewness, the sample size was big enough to satisfy this condition. Since the sample size was greater than 30, by the Central Limit Theorem, it was assumed that the distribution of sample means is approximately symmetric, making it appropriate to conduct a two-sample *t* test.

The conditions apply to both of the two-sample *t* tests as the same two samples as trials were used to collect data for both the range and deviation.

H_0: $\mu_{Ar} = \mu_{Sr}$

H_a: $\mu_{Sr} \neq \mu_{Ar}$

Figure 23. Null and Alternative Hypothesis for Two-Sample *t* Test for Range.

Figure 23 shows the two hypotheses where μ_{Ar} is the true mean range of all Accustrike darts when fired from a Nerf Eagle Strike Blaster 60 inches off the ground and μ_{Sr} is the true mean range of all Suction darts. The null hypothesis states that the mean range of all Accustrike darts would equal the mean range of all Suction darts while the alternative states that these mean ranges are not equal.

Table 2
Results of Two-Sample *t* test for Suction and Accustrike Darts

Two-Sample *t* test	
Alternate Hyp	$\mu_{Sr} \neq \mu_{Ar}$
t-statistic	5.834
P-value	$1.071 * 10^{-7}$

Table 2, above, shows the output of running the *t* test. The *t*-value of this test was 5.834 and the P-value is nearly zero which can be seen in table 2. Refer to Appendix A for calculation of *t*-statistic.

The null hypothesis was rejected because the P-value is nearly zero and much less than the alpha value of 0.05. There is convincing evidence that there exists a significant difference between the true mean range of Suction darts and Accustrike darts. There is nearly a 0% chance of getting a *t*-statistic as extreme or more than -5.77 by chance alone if H_0 is true.

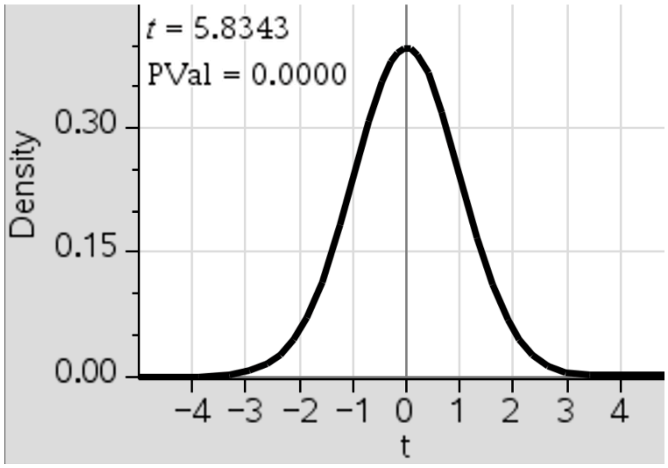


Figure 24. P-Value Density Curve for Range

Figure 24 displays the density curve of the P-value for a two-sample *t* test. The area shaded on both sides of the graph displays the P-value. The shaded area is barely visible which helps to conclude that there is a significant difference in the true mean ranges between Accustrike darts and Suction darts.

H_0: $\mu_{Ad} = \mu_{Sd}$

H_a: $\mu_{Sd} \neq \mu_{Ad}$

Figure 25. Null and Alternative Hypothesis for Two-sample *t* test for Deviation.

Figure 25 shows the two hypotheses where μ_{ad} is the true mean deviation from the centerline of all Accustrike darts when fired from a Nerf Eagle Strike Blaster 60 inches off the ground and μ_{sd} is the true mean deviation from the centerline of all Suction darts when fired from a Nerf Eagle Strike Blaster 60 inches off the ground. The null hypothesis states that the mean deviation of all Accustrike darts would equal the mean deviation of all Suction darts while the alternative states that these mean deviations are not equal.

Table 3
Results of Two-Sample *t* test for Suction and Accustrike Darts

Two-sample *t* test	
Alternate Hyp	$\mu_{Sd} \neq \mu_{Ad}$
t-statistic	-1.586
P-value	0.116

Table 3, above, shows the output of running the *t* test. The *t*-value of this test was -1.586 and the P-value is 0.116 which can be seen in figure 10.

The null hypothesis could not be rejected because the P-value of 0.116 is greater than the alpha level of 0.05. There is no convincing evidence that there exists a significant difference between the true mean deviation from the centerline of Suction darts and Accustrike darts. There is an 11.6% chance of getting a t-statistic as extreme or more than 1.57 by chance alone if H_0 is true.

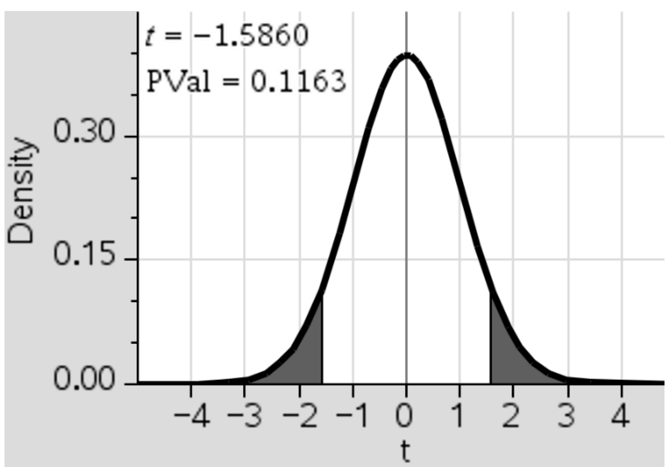


Figure 26. P-Value Density Curve for Deviation

Figure 26 displays the density curve for the P-value of the two-sample *t* test for deviations. Unlike the previous density curve, this displays a much higher chance of obtaining this *t*-statistic at least this extreme by chance alone if the true mean deviations of the Accustrike and Suction darts are equal. Therefore, it was not reasonable to assume that there was a difference in true mean deviations.

From these results, the Accustrike dart was deemed to be the most effective out of the three darts types. They traveled farther on average than the Suction darts while being far more accurate than the Normal darts. The Suction darts ended up having the shortest range while remaining relatively accurate, which follows the original hypothesis of this experiment. These findings reveal that there possibly exists a position where the center of mass can be positioned on a nerf dart so that its range and accuracy are maximized.

Conclusion

This experiment was designed and carried out to see how shifting the center of mass would affect the range and accuracy of a projectile. Nerf darts of varying masses were used as the projectiles. The Normal darts had the lightest tip and therefore the center of mass was further from the front compared to the other two darts. Accustrike darts have a heavier tip, slightly moving its center of mass forward. Suction darts' plunger-like tip was the heaviest, making its center of mass closest to the tip.

It was originally hypothesized that the projectile with a centroid that was closest to its tip, the Suction dart, would travel the shortest distance and be the most accurate. This hypothesis was accepted as the Suction dart had the shortest range out of all three darts and was more accurate than the Normal Darts.

Out of the three darts, it was determined that the Normal darts traveled the farthest with a mean range of 459.660" which was much higher than Accustrike's and Suction's mean ranges of 382.554" and 340.331" respectively. The Normal dart is the lightest out of the three darts which means that its center of mass was closer to the middle of the dart. Determining whether there was a statistical difference between the Accustrike and Suction darts' required a two-sample t-test to be conducted. A t-statistic of 5.834 was calculated, corresponding to a nearly zero P-value. Due to the low P-value, it was determined that there was a significant difference in the true mean ranges between these two darts. The Accustrike darts traveled farther on average than the Suction darts. The Accustrike dart was slightly lighter than the Suction dart and slightly heavier than the Normal dart. Therefore, its center of mass was closer to the tip when compared to the Normal dart but further from its tip when compared to the Suction dart. From these results, it was concluded that, on average, a lighter dart with a center of mass closer to its center will travel farther than a heavier dart with a center of mass closer to its tip.

Although the Normal darts traveled the furthest, they were the least accurate darts. It was concluded that the Normal darts, on average, deviated further from the centerline with a mean deviation of 18.913” which was again much higher than Accustrike’s and Suction’s deviations of 3.679” and 5.071” respectively. Another two-sample t-test was conducted to determine whether there was evidence of a significant difference in accuracy between the Accustrike and Suction darts. A t-statistic of -1.586 and a P-value of 0.116 was obtained from this statistical test. Because the P-value from this statistical test was greater than the significance level of 0.05, it was concluded that there is no evidence of a significant difference between the true mean deviations from the centerline among these two darts. Out of the three dart types, the Accustrike darts seemed to maximize the range while maintaining relatively good accuracy. They may not have traveled as far as the Normal darts, but they were far more accurate on average. The Accustrike darts also traveled further on average than the Suction darts while maintaining relatively the same accuracy.

The Normal darts traveled the farthest on average due to their lighter weight. However, the lighter weight meant that it had less stability in the air when compared to the other two darts. Less stability made it harder for the darts to travel in a straight line. The drag acting on the side of the darts caused them to fly in a different direction which resulted in poorer accuracy. In contrast, the Suction and Accustrike darts had more stability in the air due to their heavier tips which resulted in these darts having better accuracy. The Suction dart’s heavier tip might have been its downfall when it came to range. The heavier tip might have pulled down the Suction darts quicker, which is why the range was significantly smaller when compared to the other two darts.

These findings reveal that there could exist a position or range of positions for the centroid where the nerf dart could maximize range while maintaining relatively good accuracy. This range exists somewhere around where the Accustrike's center of mass lies. In essence, the mass of the tip of the dart could be manipulated in such a way as to move the center of mass to this ideal position.

Although the darts did not exactly follow a smooth parabolic path, the centroid, or the specific location of the center of mass on an object, of the darts does follow a parabolic trajectory. Newton's second law of motion states that force is the product of mass and acceleration. Projectiles launched on Earth are subject to the Earth's gravitational field where the acceleration of gravity is $9.8\, m/s^2$. During its flight, gravity is the only force acting on the projectile. In this case, $F = mg$ where g is the acceleration of gravity (What is Center of Mass). The force on the projectile will be greatest wherever the center of mass is positioned because that is where the projectile's mass is the greatest. Due to this, the center of mass traces out the projectile's trajectory, which does end up being a parabola. This phenomenon signifies that the location of the centroid must have some effect on the effectiveness of a projectile whose mass is not uniformly distributed.

A recent study by Ping Wei, a senior research scientist at NVIDIA, on super captivating missiles found results similar to that of this experiment. It is widely known that even slight changes to the mass could affect the range and accuracy of missiles. Furthermore, it is known that increasing the mass of a projectile could result in greater accuracy (Wei). These findings agree with this experiment's results as the heaviest darts, the Suction and Accustrike darts, tended to have the least amount of deviation from the centerline. Most importantly, this specific study conducted by Wei found that there indeed exists an ideal centroid position where the range can be maximized while keeping the projectile stable and thus accurate.

There was room for error when involving the mount and location of the experiment. The mount had to be reset each day in the same position. Although notes were taken on the first day on the specific measurements of the orientation of the mount, the setup error may have caused it to slightly move differently each day. This could have been solved by lining the mount up once and then taping or attaching it onto the floor. The experiment was done in a corner of the closed gym, a spot with a painted straight line. However, there was a wall off to the left of the centerline, where if a dart hit the wall, the trial was redone immediately. But some darts that deviated far to the right traveled farther to the right of the centerline than the wall is to the left of the centerline. The location of darts that hit the wall could not be tracked, but ones that flew far to the right were. Relocating to a spot with a new centerline without any nearby walls may have resulted in more outliers.

The darts themselves may have been a weakness in the experiment. Reusing the same darts for trials may wear them down and cause them to not fly the same way as when they were new. A solution to this would have been to buy enough darts for one per trial. The number of darts bought in the experiment caused each dart to be used, on average, two to three times. The three darts used in the experiment not only had different masses but also had different shaped tips. Although the purpose of the experiment is to compare darts with different centers of mass, it was not looking for the differences between the shapes of their tips. Darts with different tips or slightly different foam bases may fly differently through the air. Suction darts had a flat, plunger-like tip that had a slight inside dip, producing more drag on the dart, which is why they had the smallest range. The amount of air resistance is directly proportional to a projectile's cross-sectional area as $F_D = \frac{1}{2} C_d A p v^2$ where F_D stands for drag force, C_d stands for the drag coefficient, p stands for density, v stands for the velocity, and A stands for the cross sectional area of the projectile (The Drag Equation). But for this particular case, only the cross-sectional area was considered. Since the Suction darts had the largest cross-sectional area, it was subject to the most amount of drag compared to the other darts. This could be the reason why the Suction darts had the smallest mean range making their shape a confounding variable with its center of mass. The Normal darts had a rounded tip, which cut through the air better than the other darts and had more extreme paths. The Accustrike darts had a flat tip at the end of a small spiral. The goal of the experiment was not to compare these shapes, it was to compare their center of masses. To fix this, the tips of the darts could have been manufactured specifically for this experiment so that they have the same shape but have different masses. One way for this to be accomplished is to keep the volume of the tip constant while manipulating the density to change the mass since ($\rho V = m$) where ρ stands for density, V stands for volume, and m stands for mass.

Out of the three Nerf darts used in this experiment, the Accustrike darts seemed to be the most effective. The Accustrike darts may not have traveled as far as the Normal darts, but they were far more accurate on average. The Accustrike darts also traveled further than the Suction darts while having roughly the same deviation. In terms of making a new Nerf dart with the same foam base, the results from this experiment suggest that the dart's accuracy and range can be maximized by using a tip that is roughly the same mass as that of the Accustrike darts. It is also important to consider the aerodynamic properties of the darts as minimizing cross-sectional area would lead to minimizing the drag force and thus a greater range.

There are many ways that this research can be expanded upon. Most research on ballistics focuses on the aerodynamics of the projectile. However, accounting for the position of the centroid can add another element to ballistics research that can further aid in making projectiles more potent. There are certain aspects of this experiment that can be expanded. For example, the Nerf darts used in this experiment only had centroids closer to the tip. But many ballistics engineers deal with projectiles that have most of the mass near the rear. Other types of projectiles could have been manufactured and tested for this experiment. Another similar experiment would have been to have three types of projectiles where one had a centroid closer to its tip, another had a centroid closer to its rear, and the final one had the centroid at its center all while having the same mass. Different launching mechanisms outside of a Nerf blaster could have been used. There are still questions on whether different launch angles and initial velocities would change how the center of mass affects the flight of the projectile. There is also the question of whether the aerodynamic properties of the Nerf darts were the main reason for the change in each dart's range and accuracy. Further research into the subject of the center of mass can lead to a greater understanding of projectile motion and aerodynamics in general.

Appendix A: Sample Calculations

$$t = \frac{\bar{X}_{ar} - \bar{X}_{sr}}{\sqrt{\frac{(s_{ar})^2 + (s_{sr})^2}{n_{ar} + n_{sr}}}}$$

$$t = \frac{382.554'' - 340.332''}{\sqrt{\frac{28.298^2 + 40.105^2}{45}}}$$

Figure 27. *t* -statistic Calculations for Difference of Range Tests

Figure 27 shows the equations for the *t*-statistic for the difference of the range test; one contains the variables while the bottom one contains the actual numbers. The variable $\bar{x}_{ar}$ represents the sample mean range of the Accustrike darts while C represents the sample mean range of the suction darts. The variable s_{ar} represents the sample standard deviation of the Accustrike darts while s_{sr} represents the sample standard deviation of the suction darts. The variable n_{ar} represents the sample size of the Accustrike darts while n_{sr} represents the sample size of the suction darts.

Appendix B: Blaster Mount

Materials:

(2) Pieces of Wood (2x4x60)
Plywood (16”x 48”)
Plywood (24” x 11.5”)
Pencil
Chop Saw
Drill
(18)Screw Fasteners (6”)

Procedure:

1. Refer to safety protocols before operating any machinery and assure safety glasses are worn.

2. Grab a piece of 2x4 and measure 56.5” from one of the sides and mark with a pencil.

3. After marking, use the chop saw to cut at the 56.5” line.

4. Repeat steps 2-3 for the other piece of wood.

5. Mark a 10” line on the leftover 2x4 that was unused.

6. Cut at the 10” line and place in between the top edge of the (2) 2x4.

7. Fasten 4 screws, 2 on each side, through the 2x4 into the 10” piece. Refer to figure 11.

8. Flip over the connected piece of wood to where the 10” piece is flat on the ground.

9. Find the approximate center of the long side of the 16”x 48” plywood and mark with a pencil across.

10. Place the plywood on the 2x4, so that each leg of the stand is equidistant from the edge of the plywood.

11. Using 4 more screws, fasten from the top down through the plywood and into each leg of the 2x4. Using 2 screws on each leg.

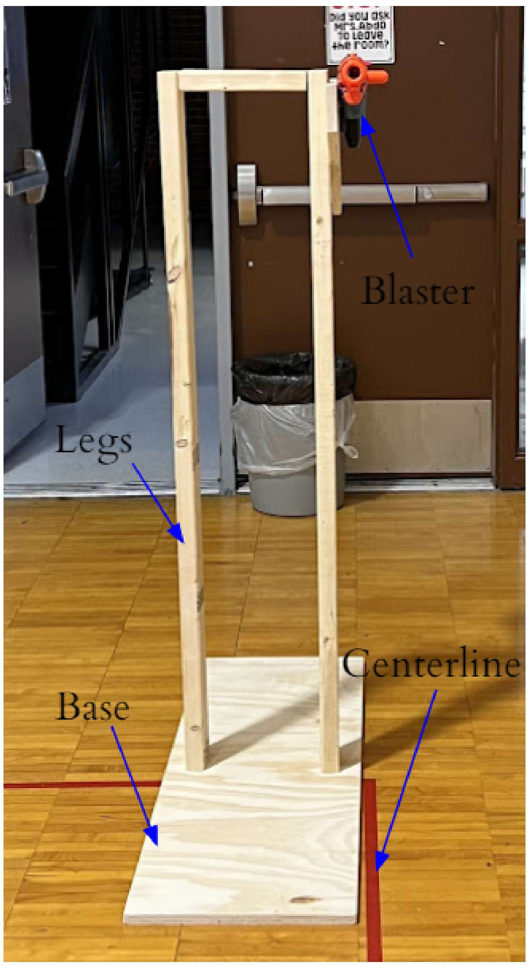
12. Carefully flip over the connected stand.

13. On a table, place the blaster on top of the 24” x 11.5” plywood, so that the longest side of the plywood and longer side of the blaster are parallel to each other.

14. Use a pencil to mark where the screws will be fastened. These marks were determined by where the blaster would be balanced. i.e through open holes of the blaster. Refer to figure 11 for clarification. Adjustments of the plywood might need to be made if the blaster is placed too far back on the plywood, blocking the nerf darts from shooting out.

15. Where previously marked with a pencil, slightly drill in screws about 1” into the wood.

16. Fasten the plywood to the rest of the stand by using screws and drilling through the plywood and fastening it into one of the legs of the stand. Use 2 screws minimum.



Works Cited

Alioto, Zion."AERODYNAMICS." *The Physics of Air Rifles*, 2019, ffden-2.phys.uaf.edu/webproj/212_spring_2019/Zion_Alioto/zion_alioto/Page3.html.

"What Is Center of Mass." *Khan Academy*, Khan Academy, 2016, https://www.khanacademy.org/science/physics/linear-momentum/center-of-mass/a/what-is-center-of-mass#:~:text=What%20is%20the%20center%20of,is%20located%20at%20the%20centroid.

Ling, Samuel J., et al. University Physics: Volume 1, 9.6 Center of Mass, OpenStax, Rice University, 2017.

NASA. "The Drag Equation." *NASA*, NASA, 13 May 2021, https://www.grc.nasa.gov/www/k-12/airplane/drageq.html.

"University Physics Volume 1." 17.8 Shock Waves | University Physics Volume 1, 3 Aug. 2016, courses.lumenlearning.com/suny-osuniversityphysics/chapter/17-8-shock -waves/

Rafferty, John P. "Ballistics." *Encyclopædia Britannica*, Encyclopædia Britannica, Inc., https://www.britannica.com/science/ballistics.

Wei, Ping, et al. "Analysis And Calculation Of The Best Center Of Mass For Supercavitating Projectiles". *Journal Of Sensors*, vol 2022, 2022, pp. 1-10. *Hindawi Limited*, doi:10.1155/2022/9521236. Accessed 20 May 2022.